

Images analysis improvement by variational segmentation in the GRASS GIS

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Outline

- 1 Signal analysis
- 2 A quick look at the math
- 3 The software
- 4 Applications
- 5 Conclusions and future work

Introduction

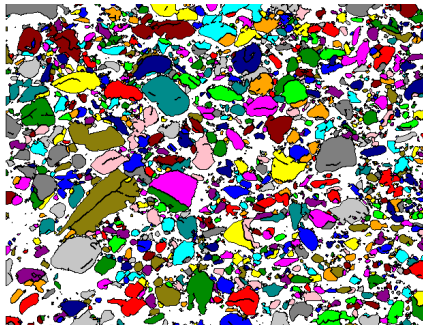
It is often useful to extract from a map or an image a "simplified" version, where uniform regions are smoothed and boundaries well defined.

One approach to this problem is to use **segmentation**.

Segmentation can be intuitively considered as the process of partitioning a domain into disjoint and homogeneous regions according to some criteria.

Introduction

An example of image segmentation



Pebbles image and its segmentation

Segmentation

The output of segmentation consists in:

- a set of homogeneous regions
- a set of boundaries

Depending on the application, one or both sets are relevant.

The problem is to identify the main signal features by smoothing the input data while preserving the signal structure (and its discontinuities).

Segmentation by variational approach

One of the possible ways to formulate the segmentation process from a mathematical viewpoint is a variational framework, which involves appropriate quantities that once minimized produce a solution directly depending on the properties of such quantities.

These quantities are penalty terms related to the features required for the solution.

The coefficients assigned to these terms are the way the user control the process.

Mumford and Shah variational model

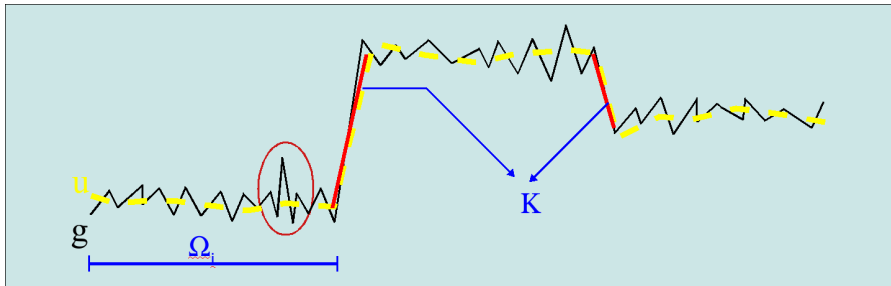
The Mumford and Shah variational model follows this approach, requiring:

- 1 the solution to be as close as possible to the input data
- 2 the solution to be as smooth as possible within each homogeneous region
- 3 the length of the regions boundaries to be as short as possible

$$MS(u, K) = \int_{\Omega \setminus K} |u - g|^2 dx + \lambda \int_{\Omega \setminus K} |\nabla u|^2 dx + \alpha H^1(K)$$

where $\Omega \subset \mathbb{R}^n$ is a bounded open set, $g \in L^{\text{inf}}(\Omega)$ is the input data, λ and α are positive parameters, H^1 is the Hausdorff measure and K the set of region boundaries.

Mumford and Shah variational model



Ω_i represents an uniform region, g is the input data, u its approximation and K the set of region boundaries.

Mumford and Shah variational model

The main issues with this formulation are:

- surface (2D) and line (1D) integrals are mixed
- the integration domain is one of the unknown of the problem (it is a free discontinuity problem)

in this form it is not possible to find a solution.

Therefore the formulation is modified so that

- only surface (2D) integrals are present (weak formulation)
- Γ convergence is used to rewrite the term controlling the discontinuities
- the system of Euler equations associated to the new formulation is written, the solution is iterative

Mumford and Shah model extension

While the new formulation makes the model implementation possible, some problems for its application to real signals remain:

- controlling smoothness by first order derivatives can yield to over-segmentation ("ramp" effect)
- discontinuities curves can only end presenting a free extremity or a triple junction
- the discontinuities can meet the domain boundary only orthogonally
- corners cannot be described

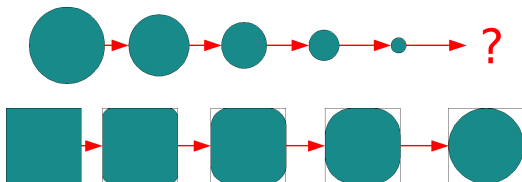
Mumford and Shah model extension

It is possible to model corners by modifying the functional

$$MS_k(u) := \int_{\Omega} |u - g|^2 dx + \lambda \int_{\Omega} |\nabla u|^2 dx + \int_{S_u} (\alpha + \beta k^2) ds$$

where k is the contour curvature. Here the parameters α and β control the elasticity and the rigidity of the curves respectively:

- if $\alpha \rightarrow 0$ each pixel tends to become a single segmented region
- if α is too large then circles collapse, squares corners are rounded out and a single segmenting region appears
- if $\lambda \rightarrow \infty$ the solution tends to be piecewise constant



The *seglib* library

A new C library has been developed to implement a set of core functions to perform image segmentation.

The library, called *seglib*, implements the original Mumford-Shah model and the Mumford-Shah model with the curvature term.

The library implements, for both models, the nonlinear Jacobi and the nonlinear Gauss-Seidel iterative methods to solve the discrete gradient descent problem associated to the finite differences discretization of the Euler equations.

The source code is released under the GNU3 General Public License.

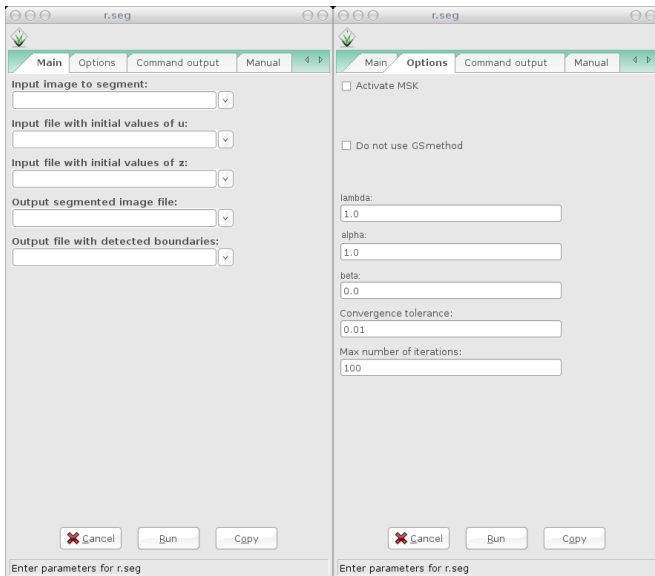
The *r.seg* GRASS module

The GRASS module *r.seg* provides an end-user program to perform image segmentation in the GRASS GIS environment by accessing the *seglib* library functions.

The choice of developing a GRASS module rather than a stand alone program depends mainly on the advantage of integrating an image analysis tool in a GIS.

The source code is released under the GNU3 General Public License.

The *r.seg* GRASS module interface



The *r.seg* GRASS module parameters I

The *r.seg* GRASS module parameters are:

- the name of the input raster map to be segmented
- the names of the output raster maps containing the segmented image and the detected boundaries
- the name of the raster maps containing the initial values for the u and z functions
- the value of the λ term, controlling the smoothness of the regions in the segmented map
- the value of the α term, controlling the length of the regions boundaries
- the value of the β term, controlling the rigidity of the regions boundaries (the *Activate MSK flag* must be set)

The *r.seg* GRASS module parameters II

- the value of the *Convergence tolerance* defining the maximum difference, over the entire domain, between the values of the solution (smoothed image) at two consecutive iteration steps
- the value of the *Max number of iterations* parameter indicates the maximum number of iterations before stopping the process (if convergence is not reached)
- the *Do not use GS method* flag deactivates the use of the Gauss-Seidel method and therefore activates the use of the Jacoby method. This option is only for test purposes.

r.seg module input maps

Two initial maps are necessary to bootstrap the iterative algorithm:

- the map for the initial values of u can be set equal to the input raster.
- the map for the initial values of z must contain values (even constant) in the $[0-1]$ range.

The maps containing initial values for z can be created easily with the *r.mapcal* GRASS module.

Testing

The *r.seg* GRASS module has been used to analyze test and some typical images used in a GIS environment, to filter noise and to preprocess images before applying classification.

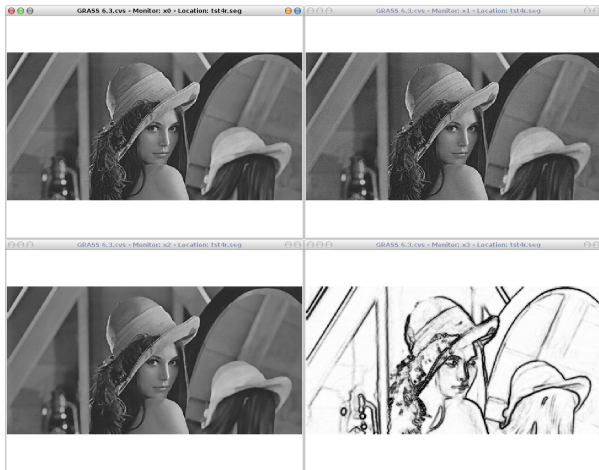
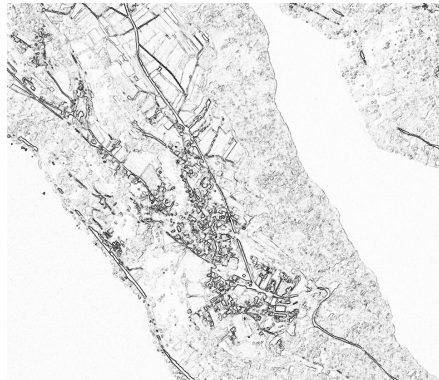


Image pre-processing



Orthophoto of the Tenna hill between the Levico and Caldonazzo lakes in Trentino (Italy), RGB bands, 1m res.

Image pre-processing



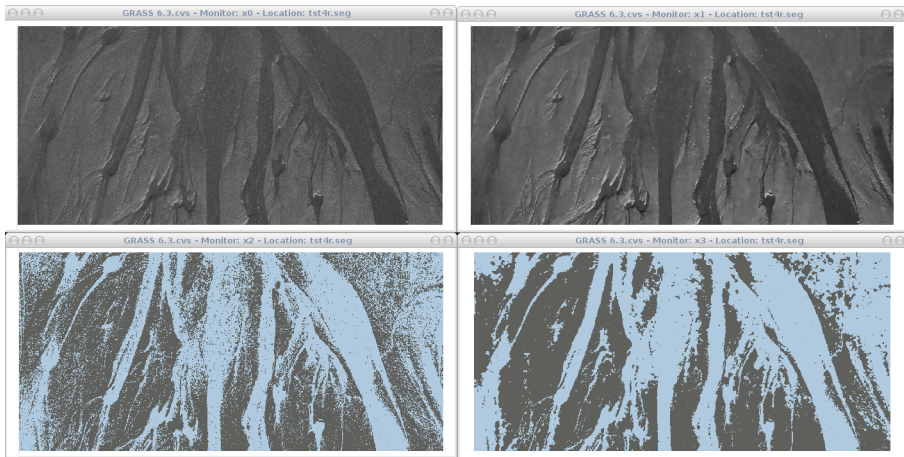
Segmented image and discontinuities.

Image classification



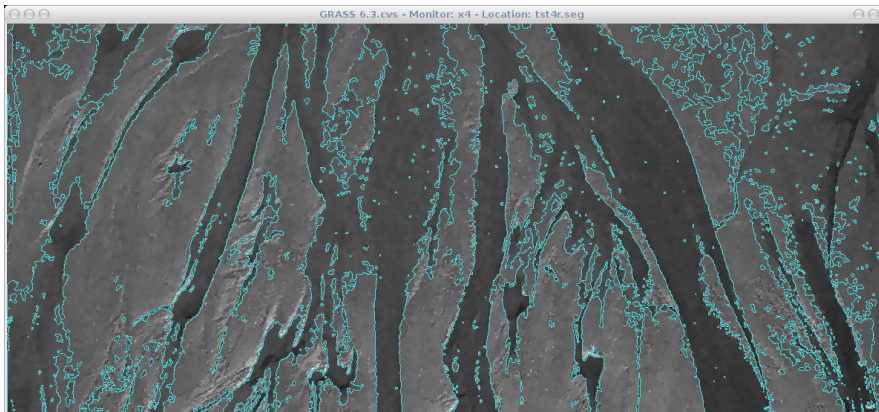
Unsupervised classification: original and segmented image.

Image classification



Segmentation and classification of the bed of a braided river. Original and segmented image, wet part in cyan.

Boundaries extraction



Boundaries extraction: the boundaries of the wet channels in cyan.

Conclusions and future work

The application of the Mumford-Shah segmentation models perform very well on images of different types. The new *seglib* library and the *r.seg* are reliable and fast.

The use of this new tool as an image pre-processor and its combination with the other procedures for image analysis in GRASS provides an evident enhancement over the standard procedure.

Numerical test to asses the benefits of this approach and to give indications on the choice of the λ , α and β parameters are under way.

Conclusions and future work

Future developments:

- direct segmentation of multi-band images
- study of the Blake and Zisserman functional (controlling the second derivatives, i.e. the curvature) in 2D for Digital Surface Models - Feature Extraction (e.g. buildings roofs)
- Mumford and Shah functional extension by the introduction of a term controlling the existence and the behavior of end points

The source code of the current development version (almost no documentation!) of the GRASS library and module is available here

`http://www.ing.unitn.it/~grass`